VISUAL PRESENTATION OF THE MOTION AND ORIENTATION OF AN ORBITING SPACECRAFT (OGO)

by Michael Mahoney and John Quann

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SUMMARY

The motion and orientation of an orbiting spacecraft are normally represented by vectors and angles in the various coordinate systems (celestial inertial coordinates, spacecraft coordinates, geodetic coordinates, etc.); and it is difficult to interpret spacecraft behavior in terms of actual orbital position and attitude. Motion and orientation information for a particular satellite (OGO) have been analyzed by the 1107 computer (Univac) and the 4020 microfilm plotter (Stromberg-Carlson), and numerical data on satellite behavior were made available. The data were then used to create a motion picture film illustrating satellite attitude in orbit.

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INTRODUCTION

The National Aeronautics and Space Administration will soon orbit the first of a series of satellites called Orbiting Geophysical Observatories (OGO), which fall into the category of "second-generation" satellites.* Satellites launched in the past have carried few scientific experiments, and the spacecraft was designed around these. Most of the early satellites were characterized by low weight, inflexible data systems, and spin-stabilization.

The era of an observatory in space is now here. The design of a spacecraft observatory may incorporate fifty or more diversified experiments, have versatile data-handling equipment to accommodate any experiment requirements, and employ sophisticated attitude-control systems. Close control of spacecraft attitude is one of the most important requirements of an orbiting observatory. The OGO (Figures 1 and 2) employs a system allowing orientation of any particular experiment with the earth, the sun, or the orbital plane.

Experiments may be mounted on any one of three sections of the spacecraft, each of which maintains a unique orientation. The main body of the satellite is oriented towards the earth; the solar paddles which contain solar-oriented experiment packages (SOEPs) are oriented toward the sun; and the orbital-plane experiment package (OPEP) is oriented into the orbital plane. The OPEP maintains its front surface into the orbital plane, using a caged gyro as a sensing element (Figure 3).

The sophisticated attitude-control system of OGO, while one of its most important characteristics, is also one of the most critical. The success of most experiments depends upon accurate spacecraft orientation, and in addition, some experiments may be rendered useless by a complete, or even a partial failure of the attitude-control system.

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*OGO I (1964 54A) was successfully launched on 5 September 1964 from Cape Kennedy.
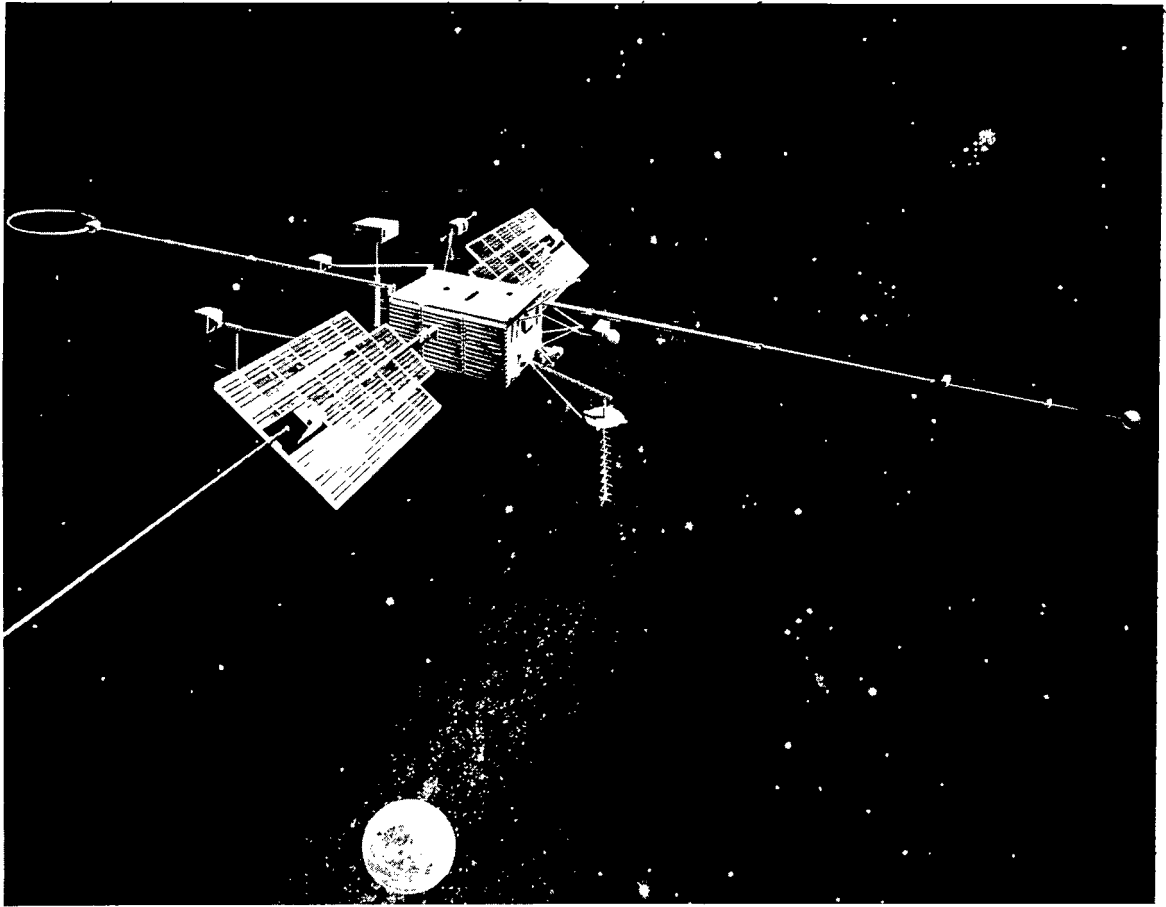
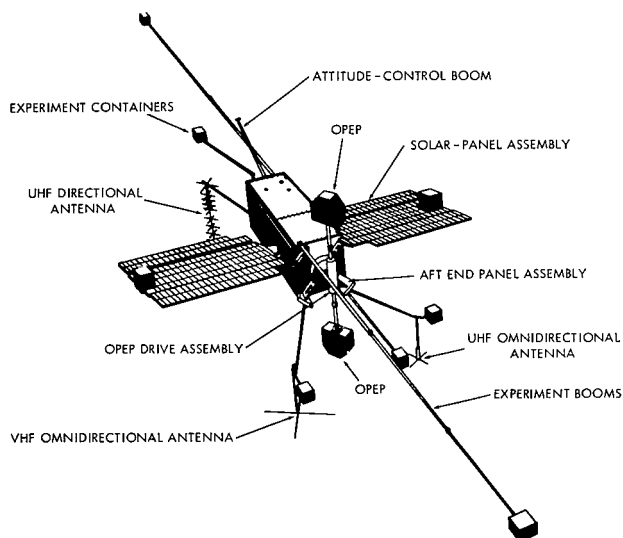Figure 1—Artist's conception of the Orbiting Geophysical Observatory.

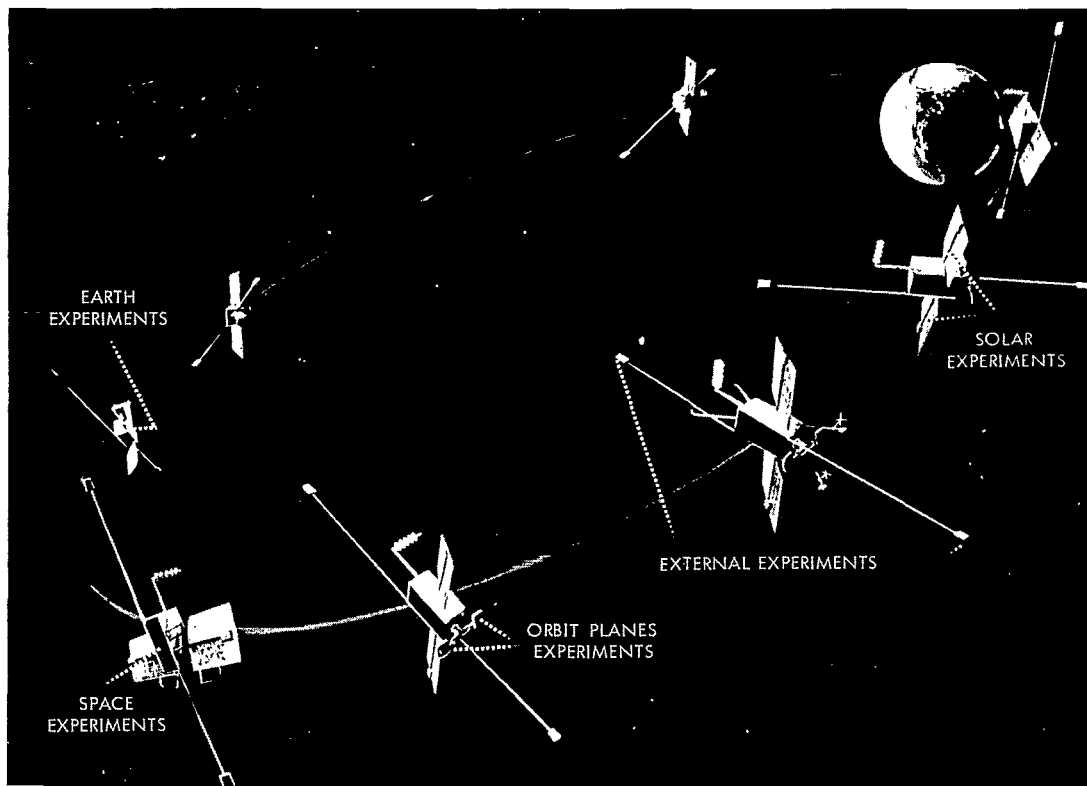Figure 2—OGO deployed configuration and antenna array.

Figure 3—OGO spacecraft orientation planes.

DESCRIPTION OF THE OGO ATTITUDE-CONTROL SYSTEM

The Orbiting Geophysical Observatory has three independent sets of spacecraft coordinates, which are maintained with respect to the body, the SOEPs and the OPEP, respectively. The body is oriented towards the earth; the SOEPs are oriented toward the sun; and the OPEP is oriented in the direction of motion of the spacecraft. The three axes of each set of coordinates are mutually perpendicular, and are defined as follows:

X_b - BODY X axis—the lateral axis through the body, along the SOEP X_p axis,

Y_b - BODY Y axis—the transverse axis through the body, through the OPEP mounting,

Z_b - BODY Z axis—the vertical axis through the body, always pointing towards the earth,

X_p - SOEP X axis—coincident with the body X_b axis. The solar paddles always rotate about this axis as they point towards the sun's disc,

Y_p - SOEP Y axis—perpendicular to the plane of the paddles, always pointing to the sun,

Z_p - SOEP Z axis—lies in the plane of the paddles, perpendicular to X_b and Y_p,

X_e - OPEP X axis—coincident with the spacecraft velocity vector,

Y_e - OPEP Y axis—perpendicular to both the X_e axis and the Z_b axis, and

Z_e - OPEP Z axis—parallel to the body Z_b axis.

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We must define two angles between these coordinate systems:

ϕ_p is the angle of rotation of the SOEP Y_p axis about the BODY X_b axis as the SOEP follows the sun, and

ψ_e is the angle of rotation of the OPEP X_e axis about the BODY Z_b axis as the OPEP follows the spacecraft velocity vector.

These nine coordinate axes and two angles can be used to determine the attitude of the three spacecraft subsystems, but their utility is limited since they are coordinate axes for the spacecraft only.

Since the attitude of the spacecraft must be presented with respect to the earth and the sun; the use of spacecraft coordinates is of limited value. This use would correspond to keeping the spacecraft fixed and having the earth and sun revolve about it. The solution is to rotate the spacecraft coordinates into celestial inertial coordinates, having the earth at the center of the system (Figure 4). The X_c axis lies in the plane of the equator and is directed towards the first point of
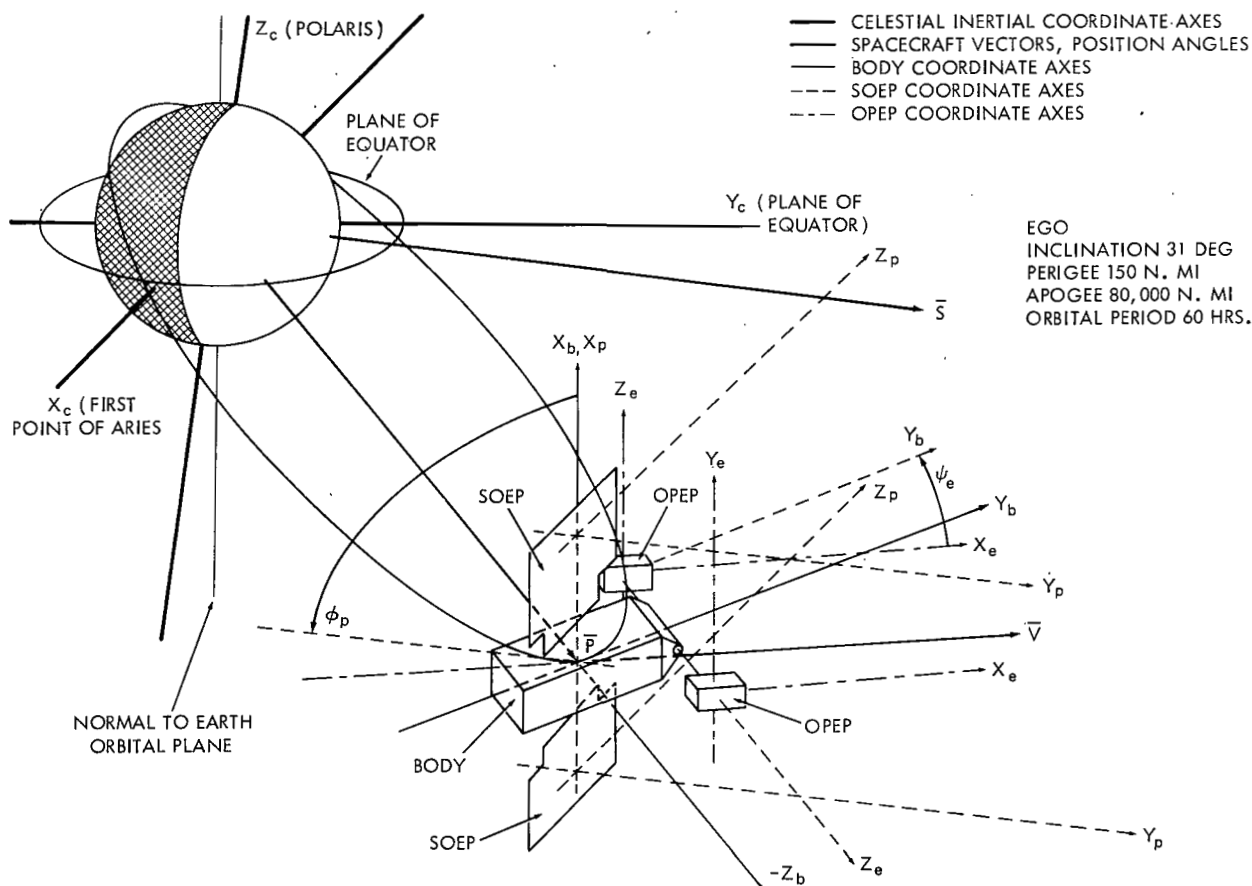
Figure 4—Spacecraft coordinates, spacecraft vectors, and celestial inertial coordinates.

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the star Aries. The Z_c axis is perpendicular to the plane of the equator, pointing towards Polaris, the north star. The Y_c axis lies in the plane of the equator such that it is mutually perpendicular to the X_c and Z_c axes.

This system, using celestial inertial coordinates, has meaning to any observer since the satellite motion and orientation are, in fact, related to the earth rather than any other reference. The system is complicated, however, by the necessity to use 27 vectors in replacing the 9 of the spacecraft coordinate system, since each of the 9 spacecraft vectors has an X, a Y, and a Z value in the new system. Tables 1 and 2 contain sample data which might be telemetered from a satellite.* Table 1 contains information on the satellite orbital position and Table 2 contains information on the conversion of orbital position data to satellite attitude data.

MATHEMATICAL ANALYSIS OF THE ATTITUDE-CONTROL SYSTEM

After defining spacecraft coordinates, we may calculate their values in the celestial inertial coordinate system. Data from the expected orbit of OGO (after it is launched) will be used in the following analysis. From this data we may obtain all the information necessary to calculate the attitude of the spacecraft. Three vectors in the celestial inertial coordinate system are of primary concern:

(1) position of the spacecraft $\overline{P}$,
(2) position of the sun $\overline{S}$, and
(3) velocity of the spacecraft $\overline{V}$.

The body Z_b axis of the observatory is oriented towards the center of the earth's disc. Therefore, the unit vector $\overline{Z}_b$ is

$$\overline{Z}_b = \frac{-\overline{P}}{|P|} \tag{1}$$

The SOEP Y_p axis points toward the center of the sun's disc, and the position of the sun $\overline{S}$ (as measured from the earth) is known. Thus the solar position vector from the spacecraft becomes

$$\frac{(\overline{S} - \overline{P})}{|\overline{S} - \overline{P}|} ,$$

a unit vector, and the unit vector $\overline{Y}_p$ is equal to the solar position vector

$$\overline{Y}_p = \frac{(\overline{S} - \overline{P})}{|\overline{S} - \overline{P}|} . \tag{2}$$

The OPEP X_e axis is always oriented in the direction of the spacecraft velocity vector; so the unit vector $\overline{X}_e$ is

*These data might also be computed from satellite tracking data.

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Table 1

Sample Orbit Information Computed for an Instant of Time.

Time of Data 183 120000.		Local Time 2 hrs 48 min 1.00 sec	
Right Ascension 41.35395		Declination -15.74462°	
Position Vector	PX	PY	PZ
Magnitude	4877. km	4293. km	-1832. km
Velocity Vector	VX	VY	VZ
Magnitude	-4.3564 km/sec	8.7097 km/sec	4.2609 km/sec
Solar Vector	SX	SY	SZ
Magnitude	-24,285,378. km	137,822,250. km	59,765,857. km
Longitude 121.77864°	Height 373.81 km	Latitude -15.74462°	
True Anomaly 13.95780°			
Sun Earth Satellite Angle 69.26325°			

Table 2

Sample Attitude Information Computed for an Instant of Time.

IDEAL MAIN BODY

Coord. Axis	XbXi	YbXi	ZbXi	XbYi	YbYi	ZbYi	XbZi	YbZi	ZbZi
Dir. Cosine	.5298	-.2571	.8082	-.4442	.7277	.5227	-.7225	-.6359	.2714

IDEAL PADDLE

Coord. Axis	XpXi	YpXi	ZpXi	XpYi	YpYi	ZpYi	XpZi	YpZi	ZpZi
Dir. Cosine	.5298	-.2571	.8082	-.1596	.9057	.3928	-.8329	-.3371	.4388

IDEAL OPEP

Coord. Axis	XeXi	YeXi	ZeXi	XeYi	YeYi	ZeYi	XeZi	YeZi	ZeZi
Dir. Cosine	-.4971	.7506	.4353	-.4805	.1796	-.8584	-.7225	-.6359	.2714

ACTUAL MAIN BODY

Coord. Axis	XbX	XbY	XbZ	YbX	YbY	YbZ	ZbX	ZbY	ZbZ
Dir. Cosine	.5298	-.2571	.8082	-.4442	.7277	.5227	-.7225	-.6359	.2714

ACTUAL PADDLE

Coord. Axis	XpX	XpY	XpZ	YpX	YpY	YpZ	ZpX	ZpY	ZpZ
Dir. Cosine	.5298	-.2571	.8082	-.1596	.9057	.3928	-.8329	-.3371	.4388

ACTUAL OPEP

Coord. Axis	XeX	XeY	XeZ	YeX	YeY	YeZ	ZeX	ZeY	ZeZ
Dir. Cosine	-.4971	.7506	.4353	-.4805	.1796	-.8584	-.7225	-.6359	.2714

$$\overline{X}_e = \frac{\overline{V}}{|V|} \tag{3}$$

$\overline{X}_b$ can be calculated from the known values of $\overline{Z}_b$ and $\overline{Y}_p$. Since

$$\cos \phi_p = \frac{\overline{Z}_b \cdot \overline{Y}_p}{|Z_b \times Y_p|} \tag{4}$$

it follows that

$$\overline{X}_b = \frac{\left(\overline{Y}_p \times \overline{Z}_b \right)}{\sin \phi_p} = \frac{\left(\overline{Y}_p \times \overline{Z}_b \right)}{\sqrt{1 - \cos^2 \phi_p}} \tag{5}$$

Now the quantities $\overline{Z}_b$, $\overline{Y}_p$, $\overline{X}_e$, and $\overline{X}_b$ have been determined, and $\overline{Y}_b$ is the only unknown body coordinate remaining. It is seen to be:

$$\overline{Y}_b = \overline{Z}_b \times \overline{X}_b . \tag{6}$$

For the SOEPs, $\overline{Y}_p$ has already been calculated, and

$$\overline{X}_p = \overline{X}_b , \tag{7}$$

so it follows that

$$\overline{Z}_p = \overline{X}_p \times \overline{Y}_p . \tag{8}$$

For the OPEP, $\overline{X}_e$ has been calculated, and

$$\overline{Z}_e \cdot \overline{Z}_b , \tag{9}$$

and from this one sees that

$$\overline{Y}_e = \overline{Z}_e \times \overline{X}_e . \tag{10}$$

Thus the actual orientation of the spacecraft can be determined from telemetered data on position vectors and the two angles between the coordinate axes, for any instant in time. The data contained in Tables 1 and 2, while accurately representing spacecraft motion and orientation, are difficult to interpret, and it is suggested that the data could be computer-reduced and presented for examination and analysis in visual form by employing a motion picture film. This technique, using an 1107 computer (Univac) and a 4020 microfilm plotter (Stromberg-Carlson) to aid in preparing the film, has been experimentally employed for the OGO, and could be extended to visually analyze the motion and attitude of a manned spacecraft such as Gemini.

PLOTTING SPACECRAFT MOTION AND ATTITUDE

The spacecraft is dimensioned in terms of the celestial inertial coordinate system, where the BODY, SOEP, and OPEP X, Y, and Z axes are collinear with the celestial coordinate system.

Every intersection (I_n) of 2 or 3 lines is positioned in terms, then, of the celestial coordinate system. These intersections or corners are assigned a position in an array, so that point I_0 (having position $X = X_0$, $Y = Y_0$, $Z = Z_0$) would be assigned:

$$\left. \begin{array}{l} Px\left(I_0\right) = X_0 \; ; \\[2mm] Py\left(I_0\right) = Y_0 \; ; \\[2mm] Pz\left(I_0\right) = Z_0 \; . \end{array} \right\} \tag{11}$$

The lines between points are defined by means of a double-subscripted array $L(J, K)$: where J is the line number, and K has the value of 1 or 2 (indicating the start and end, respectively, of a line).

Thus, $L(J, 1)$ equals the point (I_m) where the line J starts and $L(J, 2)$ equals the point (I_n) where the line J ends.

In order to describe the spacecraft, then, it is merely necessary to progress down the list of points $L(J, K)$, starting with $J = 1$, and draw lines from point $I_1 = L(J, 1)$ whose dimensions are:

$$\left. \begin{array}{l} Px\left(I_1\right) = X_1 : \\[2mm] Py\left(I_1\right) = Y_1 : \\[2mm] Pz\left(I_1\right) = Z_1 : \end{array} \right\} \tag{12}$$

to point $I_2 = L(J, 2)$ whose dimensions are:

$$\left. \begin{array}{l} Px\left(I_2\right) = X_2 : \\[2mm] Py\left(I_2\right) = Y_2 : \\[2mm] Pz\left(I_2\right) = Z_2 \; . \end{array} \right\} \tag{13}$$

Now that the technique of dimensioning and defining the spacecraft has been explained, it is possible to show how the spacecraft model is rotated and translated so as to agree with its actual orientation.

This is accomplished by means of the actual BODY, SOEP, and OPEP X, Y, and Z vectors as ·supplied by the OGO attitude-determination program in the computer.

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Spacecraft motion and attitude may be easily visualized if the spacecraft coordinates (rectangular) are converted into celestial inertial coordinates (also rectangular) by relatively simple transformation techniques for each of the three sets of axes BODY, SOEPs and OPEP). In order to convert from one rectangular coordinate system to another, direction cosines are employed; and they relate the two coordinate systems by a series of cosine functions (Figure 5).

Since the body Z_b axis is always oriented to the earth, and is coincident with the spacecraft position vector; we shall represent the transformation of this axis from spacecraft coordinates into celestial inertial coordinates by Equation 16 which is in turn defined by Equations 14 and 15. The transformation of all other coordinate axes may be performed in a similar manner.

Let the position vector $\bar{P}$ (which is expressed in spacecraft coordinates) be

Figure 5—Spacecraft direction cosines.

$$\bar{P} = \delta\left(\bar{X}_c, \bar{Y}_c, \bar{Z}_c\right) \tag{14}$$

and let the direction cosines of each component be defined by

$$\left.\begin{array}{rcl} B_{13} &=& \cos\phi \\[2mm] B_{23} &=& \cos\theta \\[2mm] B_{33} &=& \cos\psi \end{array}\right\} \tag{15}$$

Therefore the position vector $\bar{P}$ is

$$\bar{P} = \bar{X}_c \cos\phi + \bar{Y}_c \cos\theta + \bar{Z}_c \cos\psi$$

or

$$\bar{P} = B_{13}\bar{X}_c + B_{23}\bar{Y}_c + B_{33}\bar{Z}_c \, . \tag{16}$$

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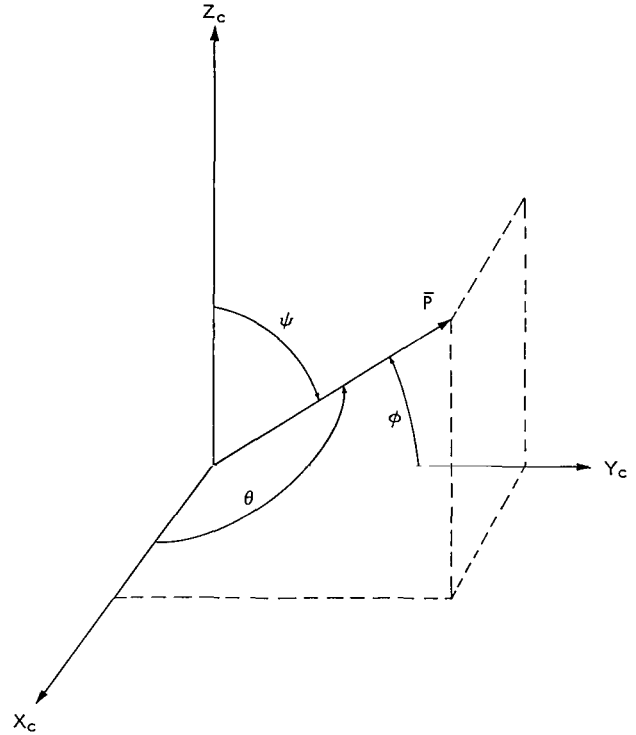This procedure, when applied to the other spacecraft coordinate axes, yields similar results.

The actual body X_b, Y_b, Z_b coordinates are defined in terms of the celestial coordinates system, where:

the X_b axis has direction cosines B_{11}, B_{12}, B_{13};

the Y_b axis has direction cosines B_{21}, B_{22}, B_{23};

the Z_b axis has direction cosines B_{31}, B_{32}, B_{33}.

Figure 6 shows how the OGO spacecraft would appear if plotted as it is defined in the computer memory using the technique just described. The points, I_b, located on the body are rotated into the celestial coordinate system, using the following transformation:

$$
\left.
\begin{aligned}
PRx\left(I_b\right) &= B_{11} X_b + B_{21} Y_b + B_{31} Z_b \\
PRy\left(I_b\right) &= B_{12} X_b + B_{22} Y_b + B_{32} Z_b \\
PRz\left(I_b\right) &= B_{13} X_b + B_{23} Y_b + B_{33} Z_b
\end{aligned}
\right\}
\tag{17}
$$

When this transformation has been completed, the OGO spacecraft as defined in the computer memory, if plotted, would appear as shown in Figure 7. The actual coordinates X_p, Y_p, Z_p of the SOEP are defined in terms of the celestial coordinate system where:

the X_p axis has direction cosines P_{11}, P_{12}, P_{13};

the Y_p axis has direction cosines P_{21}, P_{22}, P_{23};

the Z_p axis has direction cosines P_{31}, P_{32}, P_{33}.

The points I_p located on the SOEP are rotated into the celestial coordinate system using the following transformation:

$$
\left.
\begin{aligned}
PRx\left(I_p\right) &= P_{11} X_p + P_{21} Y_p + P_{31} Z_p \\
PRy\left(I_p\right) &= P_{12} X_p + P_{22} Y_p + P_{32} Z_p \\
PRz\left(I_p\right) &= P_{13} X_p + P_{23} Y_p + P_{33} Z_p
\end{aligned}
\right\}
\tag{18}
$$

When this transformation has been completed, the OGO spacecraft as defined at this point in the computer memory, when plotted in the Y, Z plane, would appear as shown in Figure 8.

The actual coordinates X_e, Y_e, Z_e of the OPEP are also defined in terms of the celestial coordinate system where:

the X_e axis has direction cosines E_{11}, E_{12}, E_{13};

the Y_e axis has direction cosines E_{21}, E_{22}, E_{23};

the Z_e axis has direction cosines E_{31}, E_{32}, E_{33}.

The OPEP system as initially defined is displaced from its actual position by the following amounts:

$$X = 0$$
$$Y = a$$
$$Z = 0$$

(19)

The points, I_e, located on the OPEP are rotated and translated into the celestial coordinate system using the following transformation:

$$PRx\left(I_e\right) = E_{11} X_e + E_{21} Y_e + E_{31} Z_e + 0$$
$$PRy\left(I_e\right) = E_{12} X_e + E_{22} Y_e + E_{32} Z_e + a$$
$$PRz\left(I_e\right) = E_{13} X_e + E_{23} Y_e + E_{33} Z_e + 0$$

(20)

This last transformation completes the task of properly orienting the satellite in space. A completed plot of this object, as defined in the computer memory and plotted in the Y, Z plane, would appear as shown in Figure 9.

Once these three transformations (Equations 18, 19, and 21) have been performed, the spacecraft can be plotted from any vantage point by means of one further transformation.

Let the vantage point be X_L, the vertical coordinate be Z_L, and Y_L defined to make a right-handed system. (In other words, the Y_L and Z_L coordinates define the plane of the plot as viewed from X_L.) Then

$$X_L \text{ has direction cosines } L_{11}, L_{12}, L_{13};$$
$$Y_L \text{ has direction cosines } L_{21}, L_{22}, L_{23};$$
$$Z_L \text{ has direction cosines } L_{31}, L_{32}, L_{33}.$$

The transformation for the entire spacecraft is:

$$PTx\left(I_L\right) = L_{11} PRx\left(I_L\right) + L_{21} PRy\left(I_L\right) + L_{31} PRz\left(I_L\right)$$
$$PTy\left(I_L\right) = L_{12} PRx\left(I_L\right) + L_{22} PRy\left(I_L\right) + L_{32} PRz\left(I_L\right)$$
$$PTz\left(I_L\right) = L_{13} PRx\left(I_L\right) + L_{23} PRy\left(I_L\right) + L_{33} PRz\left(I_L\right)$$

(21)

In the present film, the observatory is shown maintaining its orientation throughout its first few orbits. Results were based upon the satellite's achieving its nominal orbit. No attempt was made to put into the film any other features, except for a true anomaly clock which shows the
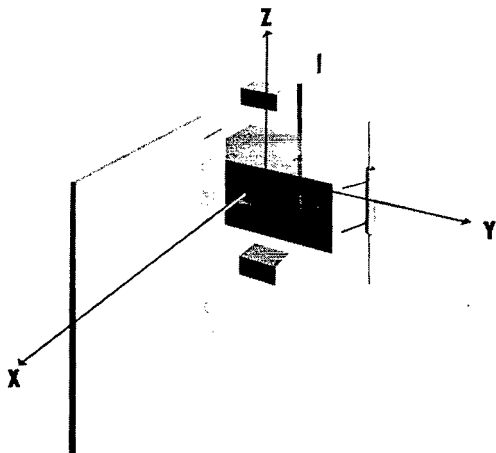
Figure 6—OGO spacecraft plotted in
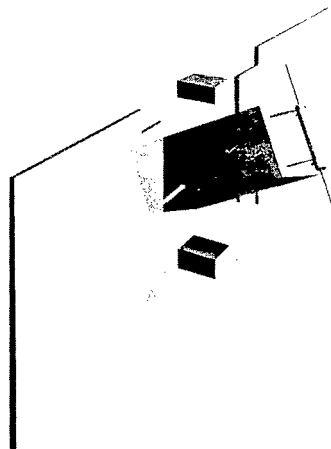spacecraft coordinate system.

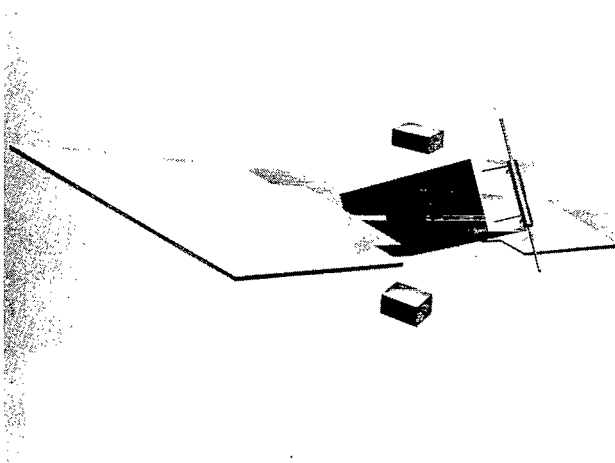Figure 7—OGO body rotated into celestial
inertial coordinates.

Figure 8—OGO solar-oriented experiment packages
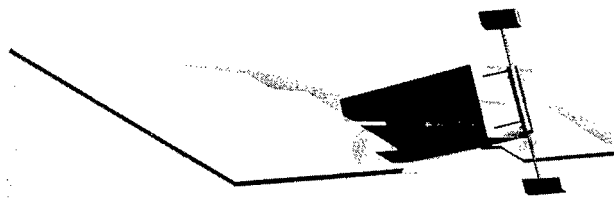(SOEPs) rotated into celestial inertial coordinates.

Figure 9—OGO orbital-plane experiment packages (OPEPs)
rotated and translated into celestial inertial coordinates.

satellite position in orbit, with respect to the perigee. Figure 10 is a sample frame from
the film.

The next step would be to incorporate the earth into the picture. Following from this, in-
clusion of the position of the sun would aid significantly in interpreting observatory behavior; and
finally, the spacecraft should be made to orbit the earth, using the proper aspect ratio of the
orbiting satellite and the earth.

CONCLUSION

The implementation of these additions has been calculated and, after the OGO has been launched, will be incorporated along with the satellite's telemetered attitude information. One use of this technique of data display is to analyze large quantities of orbit and attitude data and to determine certain effects upon the spacecraft not easily evaluated, such as: (1) the effect of darkness on the ability of the solar arrays to maintain orientation, (2) the orientation ability of the OPEP gyro at apogee (80,000 nautical miles) when the satellite velocity is at a minimum, (3) the reaction of the satellite during a yaw maneuver, and (4) the reaction of the horizon sensors, the SOEPs and the OPEP during perigee.

Since the orbital period of the OGO is more than 60 hours, the behavior of the satellite for this time can be evaluated in the 2 minutes required to display the film. In addition, this system is not limited to one spacecraft. One interesting application would be to display the attitude and motion of manned spacecraft such as Gemini or Apollo, where the capsule behavior is displayed in real time. Minor system modifications would allow pertinent system data to be simultaneously displayed, as shown by a hypothetical spacecraft display in Figure 11.

Furthermore, for a rendezvous of two orbiting spacecraft, maneuvers could be initiated prior to visual contact by means of a television link such that each pilot-astronaut could view the orientation and motion of both spacecraft simultaneously, and that the command station on the ground could view the television display.
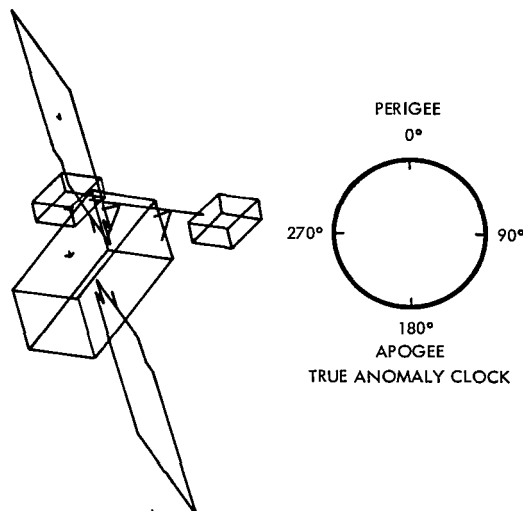

Figure 10—Sample frame from spacecraft orientation film (OGO)

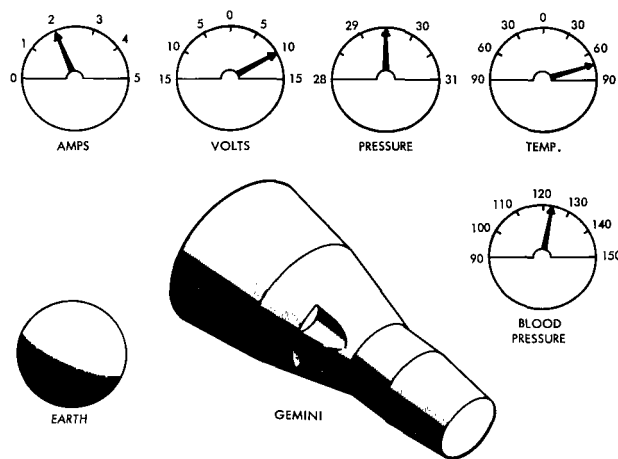

Figure 11—Frame from hypothetical orientation film used as a real-time display (Gemini)

> "The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."
>
> —NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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